

Boundary States in B -Field Background

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We consider the boundary states which describe D -branes in a constant B -field background. We show that the two-form field Φ , which interpolates commutative and non-commutative descriptions of D -branes, can be interpreted as the invariant field strength in the T-dual picture. We also show that the extended algebra parametrized by θ and Φ naturally appears as the commutation relations of the original and the T-dual coordinates.

1. Introduction

The short distance geometry seen by the extended objects such as D -branes is one of the most interesting problem in the string theory. The study of the D -brane worldvolume geometry is closely related to the study of geometry transverse to the branes in view of the Matrix Theory construction of D -branes from the lower dimensional ones. In the perturbative open string theory, the geometry on D -branes can be probed by examining the OPE algebra of vertex operators of open strings ending on the branes. Therefore, the idea of the noncommutative geometry which identifies the algebra and the geometry fits nicely with the open string description of D -branes. From the equivalent description of D -branes in the dual channel, i.e., the boundary state formalism, it is usually difficult to extract the information of the worldvolume geometry of branes. However, in some cases the boundary state formalism is more powerful than the open string approach to analyze the geometry on D -branes.

In the background of the constant metric g and the NSNS two-form field B , the algebra of open string vertex operators [1,2,3,4] and the boundary state formalism [5,6,7,8] lead to the same worldvolume geometry of D -branes, which is characterized by the relation

$$[x^i, x^j] = i\theta^{ij}. \quad (1.1)$$

In [9], Seiberg and Witten argued that the theory on D -branes in a B -field background can be described by either commutative or noncommutative Yang-Mills theories. They also proposed that there is a family of descriptions parametrized by the two-form field Φ which interpolates these two descriptions. The closed string background g, B and the open string parameters G, θ, Φ are related by

$$\frac{1}{g+B} = \theta + \frac{1}{G+\Phi}. \quad (1.2)$$

In this paper, we use the convention $2\pi\alpha' = 1$.

In [10], the extended algebra which includes (1.1) as a subalgebra was proposed. It is defined by

$$[x^i, x^j] = i\theta^{ij}, \quad [\partial_i, x^j] = \delta_i^j, \quad [\partial_i, \partial_j] = -i\Phi_{ij}. \quad (1.3)$$

From this algebra, we can construct the variables which make it manifest that the theory on D -branes in a B -field background depends only on the combination $\hat{F} + \Phi$. In this paper, we study the string theory origin of the variable ∂_i and the algebra (1.3) from

the boundary state formalism. We show that ∂_i is obtained by transforming x^i by the Λ -symmetry in the T-dual picture.

This paper is organized as follows: In section 2, we review the construction of boundary states in a B -field background. In section 3, we present three descriptions of boundary states in a constant B -field background: the path integral representation, the operator formalism and the matrix model representation. In section 4, we consider the action of T-duality on boundary states. In section 5, we show that Φ is related to the flux invariant under the Λ -symmetry in the T-dual picture. In section 6, we argue that the algebra (1.3) naturally appears by performing the Λ -transformation in the T-dual picture. Section 7 is devoted to discussions.

2. Boundary States in B Field Background

To make this paper self-contained, we start with a review of the construction of boundary states in a B -field background in the bosonic string theory.¹

2.1. Coherent State

The boundary state is a state in the Hilbert space of a closed string satisfying some boundary conditions. The worldsheet action of the closed string in the background of constant metric g_{ij} and NSNS 2-form field b_{ij} is given by

$$S_{\text{closed}} = \int d\tau \int_0^{2\pi} d\sigma \left[\frac{1}{2} g_{ij} (\partial_\tau X^i \partial_\tau X^j - \partial_\sigma X^i \partial_\sigma X^j) - b_{ij} \partial_\tau X^i \partial_\sigma X^j \right]. \quad (2.1)$$

The conjugate momentum of X^i is defined by

$$P_i = g_{ij} \partial_\tau X^j - b_{ij} \partial_\sigma X^j, \quad (2.2)$$

which satisfies $[X^i(\sigma), P_j(\sigma')] = i\delta_j^i \delta(\sigma - \sigma')$. We take the boundary of the string to be at $\tau = 0$ and parametrized by σ . In the open string picture, the boundary states correspond to the boundary conditions at $\sigma = 0$ of the open string with the action

$$S_{\text{open}} = \int d\tau \int_0^\pi d\sigma \left[\frac{1}{2} g_{ij} (\partial_\tau X^i \partial_\tau X^j - \partial_\sigma X^i \partial_\sigma X^j) + b_{ij} \partial_\tau X^i \partial_\sigma X^j \right]. \quad (2.3)$$

¹ For the boundary state in a constant B -field background in superstring theory, see e.g. [11].

The difference of the sign in front of b in (2.1) and (2.3) comes from the fact that the orientation of the worldsheet is reversed when we make the identification $(\tau_{\text{closed}}, \sigma_{\text{closed}}) = (\sigma_{\text{open}}, \tau_{\text{open}})$.

To construct the general boundary states, it is useful to introduce the coherent state $|x\rangle$ [12] which is defined by

$$X^i(\sigma)|x\rangle = x^i(\sigma)|x\rangle. \quad (2.4)$$

For example, using $|x\rangle$ the Dirichlet state $|D\rangle$ and the Neumann state $|N\rangle$ is written as

$$\begin{aligned} |D\rangle &= |x = \text{const.}\rangle, \\ |N\rangle &= \int [dx] |x\rangle = \int [dx] \exp\left(-i \int d\sigma P_i(\sigma) x^i(\sigma)\right) |x = 0\rangle, \end{aligned} \quad (2.5)$$

where $\int [dx]$ denotes the path integral over the boundary loop $x(\sigma)$. We can easily see that these states satisfy the boundary conditions

$$P_i(\sigma)|N\rangle = 0, \quad \partial_\sigma X^i(\sigma)|D\rangle = 0. \quad (2.6)$$

2.2. Gauge Fields and Λ -Symmetry

In the closed string picture, gauge fields can be incorporated in boundary states as Wilson loops:

$$\begin{aligned} |B\rangle &= \int [dx] \exp\left(i \int d\sigma A_i(x) \partial_\sigma x^i\right) |x\rangle \\ &= \int [dx] \exp\left(i \int d\sigma \left(A_i(x) \partial_\sigma x^i - P_i x^i\right)\right) |x = 0\rangle. \end{aligned} \quad (2.7)$$

This state formally satisfies the mixed boundary condition

$$(P_i - F_{ij}(X) \partial_\sigma X^j)|B\rangle = 0. \quad (2.8)$$

For the case of non-constant F_{ij} , this expression contains divergence. In this paper, we only consider the case of constant F_{ij} . From the definition of P_i (2.2), this boundary condition depends only on the combination $b + F$. This is a consequence of the so-called Λ -symmetry. This symmetry is generated by

$$U_\Lambda = \exp\left(i \int \Lambda_i(X) \partial_\sigma X^i\right). \quad (2.9)$$

Under the Λ -symmetry, P_i transforms as

$$U_\Lambda P_i U_\Lambda^{-1} = P_i - (\partial_i \Lambda_j - \partial_j \Lambda_i) \partial_\sigma X^j, \quad (2.10)$$

or

$$U_\Lambda P_i(b) U_\Lambda^{-1} = P_i(b + d\Lambda). \quad (2.11)$$

Using $U_\Lambda |x = 0\rangle = |x = 0\rangle$, we can rewrite the boundary state as

$$\begin{aligned} |B\rangle &= \int [dx] \exp \left(i \int d\sigma \left(A_i(x) \partial_\sigma x^i - P_i(b) x^i \right) \right) |x = 0\rangle \\ &= \int [dx] e^{i \int d\sigma A_i \partial_\sigma x^i} U_\Lambda^{-1} U_\Lambda e^{-i \int d\sigma P_i(b) x^i} U_\Lambda^{-1} |x = 0\rangle \\ &= \int [dx] \exp \left(i \int d\sigma \left((A_i(x) - \Lambda_i(x)) \partial_\sigma x^i - P_i(b + d\Lambda) x^i \right) \right) |x = 0\rangle. \end{aligned} \quad (2.12)$$

Therefore, the boundary state has the symmetry²

$$A \rightarrow A - \Lambda, \quad b \rightarrow b + d\Lambda \quad (2.13)$$

which leaves the combination $\mathcal{F} = b + F$ invariant.

3. Three Representations of Boundary States

The boundary state for the D -brane with constant field strength f on it is written as

$$|B\rangle = \int [dx] \exp \left(i \int d\sigma \left(\frac{1}{2} x^i f_{ij} \partial_\sigma x^j - P_i x^i \right) \right) |x = 0\rangle. \quad (3.1)$$

Note that the information of the background g and b are contained in P_i . In this section, we see that this state can be written in three equivalent ways.

3.1. Path Integral Representation

The first is the original path integral representation (3.1). From the kinetic term for x^i , we can read off the propagator of x^i to be

$$\langle x^i(\sigma) x^j(\sigma') \rangle = [(-if\partial_\sigma)^{-1}]^{ij} = \frac{i}{2} \theta^{ij} \epsilon(\sigma - \sigma'), \quad (3.2)$$

where $\epsilon(\sigma)$ is the sign function and θ is defined by

$$\theta = f^{-1}. \quad (3.3)$$

By the point splitting regularization, the equal time commutators of x^i turn out to be

$$[x^i(\sigma), x^j(\sigma)] = i\theta^{ij}. \quad (3.4)$$

This is the closed string description of the noncommutativity of the boundary coordinates of the open string.

² See [13] for the Λ -symmetry in the noncommutative description of D -brane.

3.2. Operator Representation

The second representation of (3.1) is the operator representation. By performing the Gaussian integral over x in (3.1), we obtain

$$|B\rangle = V_\theta |x=0\rangle \quad (3.5)$$

where V_θ is given by

$$V_\theta = \exp \left(-\frac{i}{4} \int d\sigma d\sigma' P_i(\sigma) \theta^{ij} \epsilon(\sigma - \sigma') P_j(\sigma') \right). \quad (3.6)$$

Note that this unitary operator V_θ has the same form as the one introduced in [14] to construct the 3-string vertex of the open string field theory in the presence of a background B -field. (See also [15].) Therefore, we call this operator the KT operator in the following. The action of KT operator on the closed string coordinate X^i is found to be

$$V_\theta X^i(\sigma) V_\theta^{-1} = X^i(\sigma) - \frac{1}{2} \int d\sigma' \epsilon(\sigma - \sigma') \theta^{ij} P_j(\sigma'), \quad (3.7)$$

and its derivative $\partial_\sigma X^i$ transforms as

$$V_\theta \partial_\sigma X^i V_\theta^{-1} = \partial_\sigma X^i - \theta^{ij} P_j. \quad (3.8)$$

Using this relation and $\partial_\sigma X^i |x=0\rangle = 0$, we can check that $|B\rangle$ in (3.5) satisfies the mixed boundary condition

$$(\partial_\sigma X^i - \theta^{ij} P_j) |B\rangle = 0 \quad (3.9)$$

which is the same as (2.8) with $F_{ij}(X) = f_{ij}$.

3.3. Matrix Representation

The third one is the matrix representation. Since the path integral in (3.1) is taken over the variable $x^i(\sigma)$ with periodic boundary condition, $|B\rangle$ can be written as a trace over the Hilbert space on which x^i satisfies the commutation relation (1.1). Using the star product

$$f \star g = f \exp \left(\frac{i}{2} \theta^{ij} \overleftarrow{\partial}_i \overrightarrow{\partial}_j \right) g, \quad (3.10)$$

$|B\rangle$ is written as

$$|B\rangle = \text{Tr} P \exp_\star \left(-i \int d\sigma P_i(\sigma) x^i \right) |x=0\rangle \quad (3.11)$$

where the product of x^i is taken by \star and P denotes the path ordering. Since $|x = 0\rangle$ satisfies the Dirichlet boundary condition for all directions, it represents a D -instanton at the origin. Therefore, the state $|B\rangle$ in the form of (3.11) can be interpreted that the matrix coordinates ϕ^i of infinitely many D -instantons have their value $\phi^i = x^i$ [5,16,17,18]. Note that this picture can also be applied to the construction of p -brane from infinitely many p' -brane with $p = p' + \text{rank } f$.

The more generic configuration of D -instantons around this classical solution can be represented in the matrix representation as

$$|B\rangle = \text{Tr } P \exp_{\star} \left(-i \int d\sigma P_i(\sigma) \phi^i(x) \right) |x = 0\rangle, \quad (3.12)$$

and in the path integral representation as

$$|B\rangle = \int [dx] \exp \left(i \int d\sigma \left(\frac{1}{2} x^i f_{ij} \partial_{\sigma} x^j - P_i \phi^i(x) \right) \right) |x = 0\rangle. \quad (3.13)$$

The noncommutative gauge field \widehat{A}_i appears as the fluctuation around the classical solution $\phi^i = x^i$ [6,19]:

$$\phi^i = x^i + \theta^{ij} \widehat{A}_j(x). \quad (3.14)$$

These matrices satisfy

$$[\phi^i, \phi^j] = -i(\theta \widehat{F} \theta - \theta)^{ij} = -i \left\{ \theta(\widehat{F} - \theta^{-1})\theta \right\}^{ij}. \quad (3.15)$$

4. T-duality of Boundary States

In this section, we consider the action of T-duality on boundary states. T-duality for closed string coordinates is given by

$$\begin{aligned} \partial_{\sigma} \widetilde{X}_i &= g_{ij} \partial_{\tau} X^j - b_{ij} \partial_{\sigma} X^j = P_i, \\ \partial_{\tau} \widetilde{X}_i &= g_{ij} \partial_{\sigma} X^j - b_{ij} \partial_{\tau} X^j. \end{aligned} \quad (4.1)$$

The closed string background for the T-dual variable are related to the original background g and b by

$$\widetilde{g} + \widetilde{b} = \frac{1}{g + b}. \quad (4.2)$$

The Dirichlet boundary state in the original variables is written as a Neumann boundary state in the T-dual picture

$$|x = 0\rangle = \int [d\widetilde{x}] |\widetilde{x}\rangle. \quad (4.3)$$

From this, the relation between the coherent state for the original and the T-dual coordinates is found to be [20]

$$|x\rangle = \exp\left(-i \int d\sigma P_i x^i\right) |x=0\rangle = \int [d\tilde{x}] \exp\left(-i \int d\sigma \partial_\sigma \tilde{x}_i x^i\right) |\tilde{x}\rangle. \quad (4.4)$$

4.1. KT Operator and Λ -Symmetry

In the T-dual picture, the KT operator has a simple meaning. By substituting $P_i = \partial_\sigma \widetilde{X}_i$ in V_θ , it becomes

$$V_\theta = \exp\left(i \int d\sigma \frac{1}{2} \widetilde{X}_i \theta^{ij} \partial_\sigma \widetilde{X}_j\right). \quad (4.5)$$

This shows that the KT operator V_θ shifts the field strength by an amount θ in the T-dual picture. In other words, V_θ can be written as a combination of Λ -symmetry and T-duality:

$$V_\theta = T U_{d^{-1}\theta} T \quad (4.6)$$

where $d^{-1}\theta$ is a one-form defined by

$$d^{-1}\theta = \frac{1}{2} \widetilde{X}_i \theta^{ij} d\widetilde{X}_j. \quad (4.7)$$

5. Φ as T-dual Flux

In [9], the two-form field Φ was introduced to interpolate the commutative and non-commutative descriptions of D -branes in a B -field background. In [21], it was argued that this degree of freedom corresponds to the freedom of splitting the flux invariant under Λ -symmetry into the gauge field strength on D -brane f and the NSNS 2-form field b . We denote this Λ -invariant flux by B , i.e.,

$$B = b + f. \quad (5.1)$$

In [21], it was shown that the open string metric G and the two-form Φ can be read off from the effective action for infinitely many D -instantons. (See also [22,23,24] for the interpretation of Φ .) The argument in [21] is as follows: The Lagrangian for D -instantons is proportional to

$$\sqrt{\det(\widetilde{g}^{ij} + \widetilde{b}^{ij} - i[\phi^i, \phi^j])}. \quad (5.2)$$

Using (3.15), this can be written as

$$\det \theta \sqrt{\det(G + \Phi + \widehat{F})} \quad (5.3)$$

with

$$\begin{aligned} G &= f^T \tilde{g} f = -f \tilde{g} f, \\ \Phi &= f^T (\tilde{b} + \theta) f = -f (\tilde{b} + \theta) f. \end{aligned} \quad (5.4)$$

From (4.2), one can show that G and Φ of this form satisfy the relation (1.2).

We can derive the same relation by using the T-duality for the boundary state which clarifies the meaning of Φ . In the T-dual picture, the boundary state (3.1) becomes

$$\begin{aligned} |B\rangle &= \int [dx] \exp \left(i \int d\sigma \left(\frac{1}{2} x^i f_{ij} \partial_\sigma x^j - P_i x^i \right) \right) |x=0\rangle \\ &= \int [dx d\tilde{x}] \exp \left(i \int d\sigma \left(\frac{1}{2} x^i f_{ij} \partial_\sigma x^j - x^i \partial_\sigma \tilde{x}_i - \tilde{P}^i \tilde{x}_i \right) \right) |\tilde{x}=0\rangle \\ &= \int [d\tilde{x}] \exp \left(i \int d\sigma \left(\frac{1}{2} \tilde{x}_i \theta^{ij} \partial_\sigma \tilde{x}_j - \tilde{P}^i \tilde{x}_i \right) \right) |\tilde{x}=0\rangle. \end{aligned} \quad (5.5)$$

From this form of boundary state, θ can be interpreted as the gauge field strength in the T-dual picture. By adding the NSNS two-form field \tilde{b} to it, the Λ -invariant flux $\tilde{\mathcal{F}}$ in the T-dual picture becomes

$$\tilde{\mathcal{F}} = \tilde{b} + \theta. \quad (5.6)$$

Since the coupling to the noncommutative gauge field has the form

$$-i \int d\sigma P_i \theta^{ij} \hat{A}_j = i \int d\sigma \hat{A}_i \theta^{ij} \partial_\sigma \widetilde{X}_j, \quad (5.7)$$

we should raise the index of \widetilde{X}_i by

$$\widetilde{X}^i = \theta^{ij} \widetilde{X}_j \quad (5.8)$$

so that the coupling (5.7) has the canonical form. By the change of variables from \widetilde{X}_i to \widetilde{X}^i , the metric \tilde{g}^{ij} and the invariant flux $\tilde{\mathcal{F}}^{ij}$ are transformed to G_{ij} and Φ_{ij} in (5.4). Therefore, Φ can be interpreted as the invariant flux $\tilde{\mathcal{F}}$ written in the coordinate $\widetilde{X}^i = \theta^{ij} \widetilde{X}_j$:

$$\Phi = f^T \tilde{\mathcal{F}} f. \quad (5.9)$$

Note that Φ itself is not Λ -invariant while $\tilde{\mathcal{F}}$ is, since f is not invariant under the Λ -symmetry.

6. Origin of Extended Algebra

In this section, we consider the origin of the algebra (1.3) from the viewpoint of the boundary state formalism. In the following discussion, it is convenient to introduce the variable \widetilde{x}^i by

$$\widetilde{x}^i = i\theta^{ij}\partial_j. \quad (6.1)$$

In terms of x^i and \widetilde{x}^i , the algebra (1.3) is written as

$$[x^i, x^j] = i\theta^{ij}, \quad [\widetilde{x}^i, x^j] = i\theta^{ij}, \quad [\widetilde{x}^i, \widetilde{x}^j] = i\widetilde{\mathcal{F}}^{ij}. \quad (6.2)$$

6.1. Algebra from Boundary State Formalism

Using x^i and \widetilde{x}^i , we can write the Lagrangian (5.2) as

$$\sqrt{\det(\widetilde{g}^{ij} + \widetilde{b}^{ij} - i[\phi^i, \phi^j])} = \sqrt{\det(\widetilde{g}^{ij} - i[\widetilde{\phi}^i, \widetilde{\phi}^j])} \quad (6.3)$$

where $\widetilde{\phi}^i$ is given by

$$\widetilde{\phi}^i = \widetilde{x}^i + \theta^{ij}\widehat{A}_j(x). \quad (6.4)$$

These variables satisfy the relation

$$[\widetilde{\phi}^i, \widetilde{\phi}^j] = -i(\theta\widehat{F}\theta - \widetilde{\mathcal{F}})^{ij} = -i\left\{\theta(\widehat{F} + \Phi)\theta\right\}^{ij}. \quad (6.5)$$

Changing from the variable ϕ^i to $\widetilde{\phi}^i$ is equivalent to set $\widetilde{b} = 0$ by the Λ -symmetry. Since this Λ -symmetry is performed in the T-dual picture, we can guess that \widetilde{x}^i is given by

$$\widetilde{x}^i = TU_{-d-1\widetilde{b}}T(x^i). \quad (6.6)$$

In the following, using the path integral representation of boundary states we show that the variables defined by this relation satisfy the algebra (6.2). Following (6.6), we first consider the T-dual of the state (3.13):

$$\begin{aligned} |B\rangle &= \int [dx] \exp\left(i \int d\sigma \left(\frac{1}{2}x^i f_{ij} \partial_\sigma x^j - P_i(x^i + \theta^{ij}\widehat{A}_j)\right)\right) |x=0\rangle \\ &= \int [dx d\widetilde{x}] \exp\left(i \int d\sigma \left(\frac{1}{2}x^i f_{ij} \partial_\sigma x^j - \partial_\sigma \widetilde{x}_i (x^i + \theta^{ij}\widehat{A}_j) - \widetilde{P}^i \widetilde{x}_i\right)\right) |\widetilde{x}=0\rangle. \end{aligned} \quad (6.7)$$

Next we perform the Λ -transformation in order to set $\tilde{b} = 0$ in \tilde{P}^i :

$$|B\rangle = \int [dx d\tilde{x}] \exp \left(i \int d\sigma \left(\frac{1}{2} x^i f_{ij} \partial_\sigma x^j - \partial_\sigma \tilde{x}_i (x^i + \theta^{ij} \hat{A}_j) \right. \right. \\ \left. \left. + \frac{1}{2} \tilde{x}_i \tilde{b}^{ij} \partial_\sigma \tilde{x}_j - \tilde{P}^i(\tilde{b} = 0) \tilde{x}_i \right) \right) |\tilde{x} = 0\rangle. \quad (6.8)$$

Taking the T-duality one more time, the boundary state becomes

$$|B\rangle = \int [dx d\tilde{x} d\tilde{\tilde{x}}] \exp \left(i \int d\sigma \left(\frac{1}{2} x^i f_{ij} \partial_\sigma x^j - \partial_\sigma \tilde{x}_i (x^i + \theta^{ij} \hat{A}_j) \right. \right. \\ \left. \left. + \frac{1}{2} \tilde{x}_i \tilde{b}^{ij} \partial_\sigma \tilde{x}_j - \tilde{x}_i \partial_\sigma \tilde{\tilde{x}}^i - \tilde{\tilde{P}}_i \tilde{\tilde{x}}^i \right) \right) |\tilde{\tilde{x}} = 0\rangle. \quad (6.9)$$

By integrating out the variable $\tilde{\tilde{x}}^i$, we arrive at the final expression:

$$|B\rangle = \int [dx d\tilde{\tilde{x}}] \exp \left(i \int d\sigma \left(\frac{1}{2} x^i f_{ij} \partial_\sigma x^j + \frac{1}{2} (\tilde{\tilde{x}}^i - x^i - \theta^{ik} \hat{A}_k) (\tilde{b}^{-1})_{ij} \partial_\sigma (\tilde{\tilde{x}}^j - x^j - \theta^{jl} \hat{A}_l) \right. \right. \\ \left. \left. - \tilde{\tilde{P}}_i \tilde{\tilde{x}}^i \right) \right) |\tilde{\tilde{x}} = 0\rangle \\ = \int [dx d\tilde{\tilde{x}}] \exp \left(i \int d\sigma \left(\frac{1}{2} x^i f_{ij} \partial_\sigma x^j + \frac{1}{2} (\tilde{\tilde{x}}^i - x^i) (\tilde{b}^{-1})_{ij} \partial_\sigma (\tilde{\tilde{x}}^j - x^j) \right. \right. \\ \left. \left. - \tilde{\tilde{P}}_i (\tilde{\tilde{x}}^i + \theta^{ij} \hat{A}_j) \right) \right) |\tilde{\tilde{x}} = 0\rangle. \quad (6.10)$$

In the last step, we shifted the integration variable $\tilde{\tilde{x}}^i$ to $\tilde{\tilde{x}}^i + \theta^{ij} \hat{A}_j$. From this expression of $|B\rangle$, the commutation relations of x^i and $\tilde{\tilde{x}}^i$ are found to be

$$[x^i, x^j] = i\theta^{ij}, \quad [\tilde{\tilde{x}}^i - x^i, x^j] = 0, \quad [\tilde{\tilde{x}}^i - x^i, \tilde{\tilde{x}}^j - x^j] = i\tilde{b}^{ij}. \quad (6.11)$$

We can see that these relations are equivalent to (6.2). In this form of $|B\rangle$, $\tilde{\tilde{\phi}}^i$ in (6.4) naturally appear as the matrix coordinates of D -instantons in the $\tilde{\tilde{x}}^i$ frame.

6.2. Relation between x^i and $\tilde{\tilde{x}}^i$

In this subsection, we show that the variable $\tilde{\tilde{x}}^i$ appeared in the boundary state (6.10) actually satisfies the relation (6.6). In the operator formalism, this relation can be rephrased as $\tilde{\tilde{X}}^i$ being the transformation of X^i by the KT operator:

$$\tilde{\tilde{X}}^i(\sigma) = V_b^{-1} X^i(\sigma) V_b = X^i(\sigma) + \frac{1}{2} \int d\sigma' \epsilon(\sigma - \sigma') \tilde{b}^{ij} P_j(\sigma'). \quad (6.12)$$

To derive this relation from the boundary state formalism, let us consider the identity

$$0 = \int [dx d\tilde{x}] \frac{\delta}{\delta \tilde{x}^i(\sigma)} \exp \left(i \int \frac{1}{2} x^i f_{ij} \partial_\sigma x^j + \frac{1}{2} (\tilde{x}^i - x^i) (\tilde{b}^{-1})_{ij} \partial_\sigma (\tilde{x}^j - x^j) - \tilde{P}_i (\tilde{x}^i + \theta^{ij} \hat{A}_j) \right) | \tilde{x} = 0 \rangle. \quad (6.13)$$

From this identity we obtain

$$\left[\partial_\sigma (\tilde{X}^i - X^i) - \tilde{b}^{ij} \tilde{P}_j \right] | B \rangle = 0, \quad (6.14)$$

which is equivalent to (6.12) since

$$P_i = \partial_\sigma \tilde{X}_i = \tilde{P}_i. \quad (6.15)$$

From the boundary condition for X (3.9), the boundary condition for \tilde{X} becomes

$$(\partial_\sigma \tilde{X}^i - \tilde{\mathcal{F}}^{ij} P_j) | B \rangle = 0. \quad (6.16)$$

Note that \tilde{x}^i is identical to x^i when $\tilde{b} = 0$, or $b = 0$. As was pointed out in [10], this case corresponds to $\Phi = -\theta^{-1}$.

7. Discussions

In this paper, we derived the extended algebra (1.3) parametrized by θ and Φ from the boundary state formalism. We identified the extra variable ∂_i as the KT transform of the original variable x^i , i.e., the Λ -transform of x^i in the T-dual picture. This algebra may shed light on the background independent description of the dynamics of D -branes.

Although we can construct the extended phase space (x^i, \tilde{x}^i) in the closed string picture, its interpretation in the open string channel is not clear. However, we believe that this extended algebra can be constructed by the variables in the open string theory.

We comment on the relation between our interpretation of Φ and the gauge fixing of worldvolume diffeomorphism on Dp -branes [5,6,19]. Before gauge fixing, the dynamical fields on a Dp -brane are the gauge field A_i and the scalar fields ϕ^i . After fixing the gauge field to have a constant field strength $A = d^{-1}f$, the fluctuation of ϕ^i around the static gauge configuration can be identified with the noncommutative gauge field. Then the residual diffeomorphism Diff_f which preserves f corresponds to the noncommutative

gauge symmetry. On the other hand, when we fix ϕ^i to the static gauge configuration, the fluctuation of A_i around $d^{-1}f$ becomes the commutative gauge field. The author of [23] argued that the diffeomorphism which changes the value of f , $\text{Diff}/\text{Diff}_f$, corresponds to the degree of freedom for Φ .

In our discussion of Φ , we always use $A = d^{-1}f$ gauge, so we are considering the noncommutative picture. Note that the commutative picture is singular in our formalism since $\theta = 0$ corresponds to $f = \theta^{-1} = \infty$. While preserving the property that the field strength is constant, we can change the value of f by using the Λ -symmetry instead of the diffeomorphism. In [21], this degree of freedom is identified with Φ . Using this interpretation of Φ , we explicitly performed the Λ -symmetry on the boundary state and identified Φ within this formalism.

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